CBCS SCHEME

USN | | | | | | | | | | | 18MAT41

Fourth Semester B.E. Degree Examination, Jan./Feb. 2021
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Shat that $W = \log Z, Z \neq 0$ is analytic and hence find $\frac{dw}{dz}$. **(06 Marks)**

b. Derive Cauchy – Riemann equation in Cartesian coordinates. **(07 Marks)**

c. Find the analytic function $f(z) = u + iv$ given $v = e^{-x}[x\cos y + y\sin y]$. **(07 Marks)**

OR

2 a. Show that an analytic function with constant modulus is constant. **(06 Marks)**

b. If $f(z) = u + iv$ is analytic prove that

$$\left[\frac{\partial|f(z)|}{\partial x}\right]^2 + \left[\frac{\partial|f(z)|}{\partial y}\right]^2 = |f'(z)|^2.$$ **(07 Marks)**

c. If $u - v = (x-y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is analytic function, if $z = x + iy$, find $f(z)$ in terms of z. **(07 Marks)**

Module-2

3 a. State and prove Cauchy's Integral formula. **(06 Marks)**

b. Discuss the transformation $W = e^z$. **(07 Marks)**

c. Find the Bilinear transformation which sends points $Z = 0, 1, \infty$ into the points $W = -5, -1, 3$ respectively. What are the invariant points in this transformation? **(07 Marks)**

OR

4 a. Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the line i) $y = x$ ii) $y = x^2$. **(06 Marks)**

b. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $|z| = 3$. **(07 Marks)**

c. Find the Bilinear transformation that maps the points $Z = -1, i, 1$ onto the points $W = 1, i, -1$. Also find the invariant points. **(07 Marks)**

Module-3

5 a. A random variable X has the following probability function :

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find i) the value of k ii) $P(x < 6)$ iii) $P(x \geq 6)$ iv) $P(3 < x \leq 6)$. **(06 Marks)**

b. The probability that a pen manufactured by a company be defective is $\frac{1}{10}$. If 12 such pens are manufactured, what is the probability that i) Exactly 2 are defective ii) at least 2 are defective iii) none of them are defective. **(07 Marks)**

c. A sample of 100 battery cells tested to find the length of life produced the following results $\bar{x} = 12$ hours, $\sigma = 3$ hours. Assuming the data to be normally distributed what percentage of battery cells are expected to have life. i) more than 15 hours ii) less than 6 hours iii) between 10 and 14 hours. [A(1) = 0.3413, A(2) = 0.4772, A(0.67) = 0.2487]. **(07 Marks)**

[redacted]

c. The nine item of a sample have the following values 45 , 47 , 50 , 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5? **(07 Marks)**

OR

10 a. Define the following terms :
i) Null hypothesis ii) Type – I and Type II error iii) Level of significance. **(06 Marks)**

b. Ten individuals are choosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. (Given $t_{0.05} = 2.262$ for gdf). **(07 Marks)**

c. The theory predicts the proportion of beans in the four groups G1, G2, G3, G4 should be in the ratio 9:3:3:1. In an experiment with 1600 beans the numbers in the four group were 882 , 313 , 287 and 118. Does the experimental result support the theory (at 5% level of significance for 3)? **(07 Marks)**

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